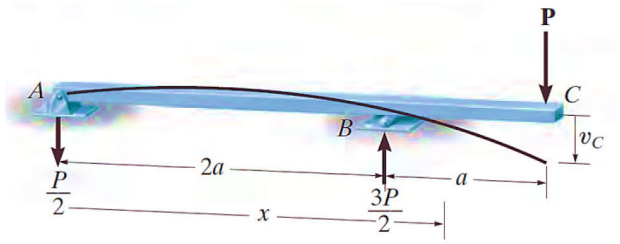


(a)

Solution 2
Moment Functions.

$$M = -\frac{P}{2}x + \frac{3P}{2}\langle x - 2a \rangle$$

Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = -\frac{P}{2}x + \frac{3P}{2}\langle x - 2a \rangle$$

$$EI\frac{dv}{dx} = -\frac{P}{4}x^2 + \frac{3P}{4}\langle x - 2a \rangle^2 + C_1$$

$$EIv = -\frac{P}{12}x^3 + \frac{3P}{12}\langle x - 2a \rangle^3 + C_1 x + C_2$$

$v = 0$ at $x = 0$,

$$0 = -\frac{P}{12}(0)^3 + 0 + C_1(0) + C_2 \Rightarrow C_2 = 0$$

$v = 0$ at $x = 2a$

$$0 = -\frac{P}{12}(2a)^3 + \frac{3P}{12}\langle 2a - 2a \rangle^3 + C_1(2a)$$

$$0 = -\frac{8Pa^3}{12} + C_1(2a) \Rightarrow C_1 = \frac{1}{3}Pa^2$$

$$EIv = -\frac{P}{12}x^3 + \frac{3P}{12}\langle x - 2a \rangle^3 + \frac{1}{3}Pa^2 x$$

The displacement at C is determined by setting $x = 3a$ We get

$$EIv_C = -\frac{P}{12}(3a)^3 + \frac{3P}{12}\langle 3a - 2a \rangle^3 + \frac{1}{3}Pa^2(3a)$$

$$EIv_C = -\frac{27Pa^3}{12} + \frac{3Pa^3}{12} + Pa^3$$

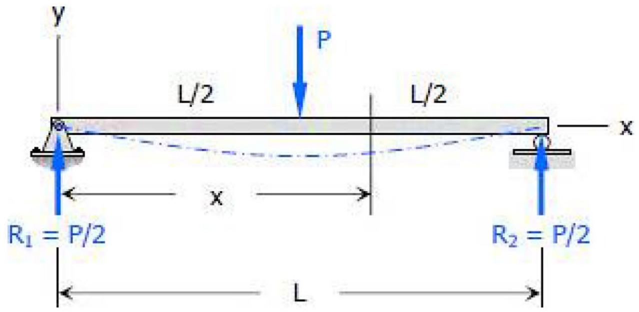
$$EIv_C = -\frac{Pa^3}{12} + \frac{3Pa^3}{12} + \frac{12}{12}Pa^3$$

$$EIv_C = -\frac{12Pa^3}{12}$$

$$\therefore \; v_C = -\frac{Pa^3}{EI}$$

EXAMPLE 5.3.4

Determine the maximum deflection δ in a simply supported beam of length L carrying a concentrated load P at midspan.



Solution

$$EIv'' = \frac{1}{2}Px - P\left\langle x - \frac{1}{2}L \right\rangle$$

$$EIv' = \frac{1}{4}Px^2 - \frac{1}{2}P\left\langle x - \frac{1}{2}L \right\rangle^2 + C_1$$

$$EIv = \frac{1}{12}Px^3 - \frac{1}{6}P\left\langle x - \frac{1}{2}L \right\rangle^3 + C_1x + C_2$$

$At\ x = 0,\ v = 0$, therefore, $C_2 = 0$

$At\ x = L,\ v = 0$

$$0 = \frac{1}{12}PL^3 - \frac{1}{6}P\left\langle L - \frac{1}{2}L \right\rangle^3 + C_1L$$

$$0 = \frac{1}{12}PL^3 - \frac{1}{48}PL^3 + C_1L$$

$$C_1 = -\frac{1}{16}PL^2$$

Thus,

$$EIv = \frac{1}{12}Px^3 - \frac{1}{6}P\left\langle x - \frac{1}{2}L \right\rangle^3 - \frac{1}{16}PL^2x$$

Maximum deflection will occur at $x = \frac{1}{2}L\ (midspan)$

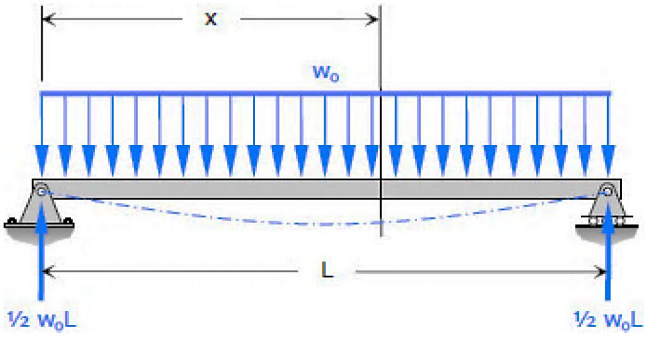
$$EIv_{max} = \frac{1}{12}P\left(\frac{1}{2}L\right)^3 - \frac{1}{6}P\left(\frac{1}{2}L - \frac{1}{2}L\right)^3 - \frac{1}{16}PL^2\left(\frac{1}{2}L\right)$$

$$EIv_{max} = \frac{1}{96}PL^3 - 0 - \frac{1}{32}PL^3$$

$$v_{max} = -\frac{PL^3}{48EI}\quad Ans.$$

EXAMPLE 5.3.5

Determine the maximum deflection v in a simply supported beam of length L carrying a uniformly distributed load of intensity w_o applied over its entire length.



Solution

$$EIv'' = \frac{1}{2}w_o Lx - w_o x\left(\frac{1}{2}x\right)$$

$$EIv'' = \frac{1}{2}w_o Lx - \frac{1}{2}w_o x^2$$

$$EIv' = \frac{1}{4}w_o Lx^2 - \frac{1}{6}w_o x^3 + C_1$$

$$EIv = \frac{1}{12}w_o Lx^3 - \frac{1}{24}w_o x^4 + C_1 x + C_2$$

$At\ x = 0,\ v = 0$, therefore $C_2 = 0$

$At\ x = L,\ v = 0$

$$0 = \frac{1}{12}w_o L^4 - \frac{1}{24}w_o L^4 + C_1 L$$

$$C_1 = -\frac{1}{24}w_o L^3$$

Therefore,

$$EIv = \frac{1}{12}w_o Lx^3 - \frac{1}{24}w_o x^4 - \frac{1}{24}w_o L^3 x$$

Maximum deflection will occur at $x = \frac{1}{2}L\ (midspan)$

$$EIv_{max} = \frac{1}{12}w_o L\left(\frac{1}{2}L\right)^3 - \frac{1}{24}w_o\left(\frac{1}{2}L\right)^4 - \frac{1}{24}w_o L^3\left(\frac{1}{2}L\right)$$

$$EIv_{max} = \frac{1}{96}w_o L^4 - \frac{1}{384}w_o L^4 - \frac{1}{48}w_o L^4$$

$$EIv_{max} = -\frac{5}{384}w_o L^4$$

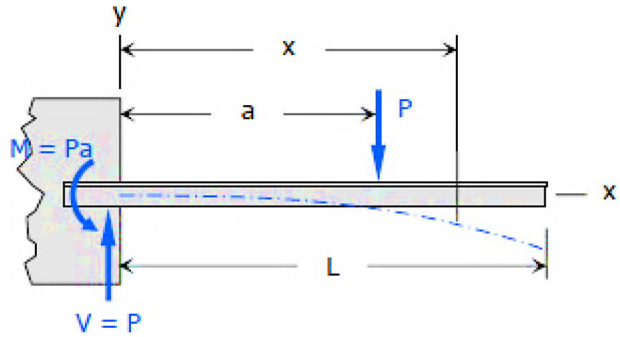
$$v_{max} = -\frac{5w_o L^4}{384EI} \qquad Ans.$$

From **Example 5.3.1** $\boldsymbol{W_0 = 4}$ **kN/m** and $\boldsymbol{L = 10}$ **m**

$$v_{max} = -\frac{5w_o L^4}{384EI} = -\frac{5(4)(10)^4}{384EI} = \frac{520.8333}{EI} \cong -\frac{521}{EI}$$

EXAMPLE 5.3.6

Determine the maximum deflection v for the cantilever beam loaded as shown in the Figure.



Solution

$$EIv'' = -Pa + Px - P\langle x - a\rangle$$

$$EIv' = -Pax + \frac{1}{2}Px^2 - \frac{1}{2}P\langle x - a\rangle^2 + C_1$$

$$EIv = -\frac{1}{2}Pax^2 + \frac{1}{6}Px^3 - \frac{1}{6}P\langle x - a\rangle^3 + C_1 x + C_2$$

$At\ x = 0,\ v' = 0$, therefore $C_1 = 0$

$At\ x = 0,\ v = 0$, therefore $C_2 = 0$

Therefore,

$$EIv = -\frac{1}{2}Pax^2 + \frac{1}{6}Px^3 - \frac{1}{6}P\langle x - a\rangle^3$$

The maximum value of EIv is at $x = L\ (free\ end)$

$$EIv_{\max} = -\frac{1}{2}PaL^2 + \frac{1}{6}PL^3 - \frac{1}{6}P(L-a)^3$$

$$EIv_{\max} = -\frac{1}{2}PaL^2 + \frac{1}{6}PL^3 - \frac{1}{6}P(L^3 - 3L^2a + 3La^2 - a^3)$$

$$EIv_{\max} = -\frac{1}{2}PaL^2 + \frac{1}{6}PL^3 - \frac{1}{6}PL^3 + \frac{1}{2}PL^2a - \frac{1}{2}PLa^2 + \frac{1}{6}Pa^3$$

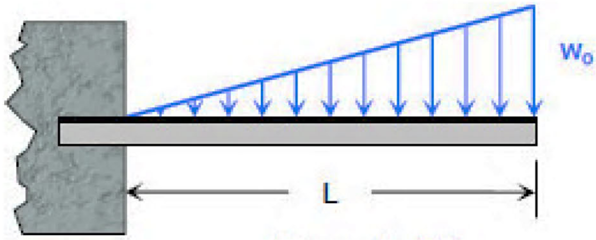
$$EIv_{\max} = -\frac{1}{2}PLa^2 + \frac{1}{6}Pa^3$$

$$EIv_{\max} = -\frac{1}{6}Pa^2(3L - a)$$

$$v_{\max} = -\frac{Pa^2}{6EI}(3L - a) \quad Ans.$$

EXAMPLE 5.3.7

Find the equation of the elastic curve for the cantilever beam shown in the Figure; it carries a load that varies from zero at the wall to w_o at the free end. Take the origin at the wall.



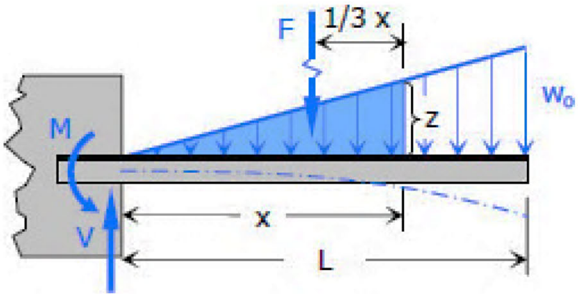
Solution

$$V = \frac{1}{2} w_o L$$

$$M = \frac{1}{2} w_o L \left(\frac{2}{3} L\right)$$

$$M = \frac{1}{3} w_o L^2$$

By ratio and proportion



$$\frac{z}{x} = \frac{w_o}{L}$$

$$z = \frac{w_o}{L} x$$

$$F = \frac{1}{2} x z$$

$$F = \frac{1}{2} x \left(\frac{w_o}{L} x\right)$$

$$F = \frac{w_o}{2L} x^2$$

$$EIv'' = -M + Vx - F\left(\frac{1}{3} x\right)$$

$$EIv'' = -\frac{1}{3} w_o L^2 + \frac{1}{2} w_o L x - \frac{1}{3} x \left(\frac{w_o}{2L} x^2\right)$$

$$EIv'' = -\frac{w_o L^2}{3} + \frac{w_o L}{2} x - \frac{w_o}{6L} x^3$$

$$EIv' = -\frac{w_o L^2}{3} x + \frac{w_o L}{4} x^2 - \frac{w_o}{24L} x^4 + C_1$$

$$EIv = -\frac{w_o L^2}{6} x^2 + \frac{w_o L}{12} x^3 - \frac{w_o}{120L} x^5 + C_1 x + C_2$$

At $x = 0,\ v' = 0$, therefore $C_1 = 0$

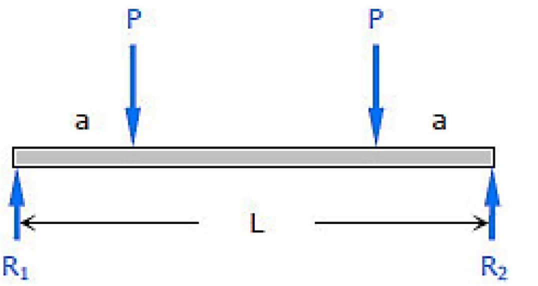
At $x = 0,\ v = 0$, therefore $C_2 = 0$

Therefore, the equation of the elastic curve is

$$EIv = -\frac{w_o L^2}{6} x^2 + \frac{w_o L}{12} x^3 - \frac{w_o}{120L} x^5 \qquad Ans.$$

EXAMPLE 5.3.8

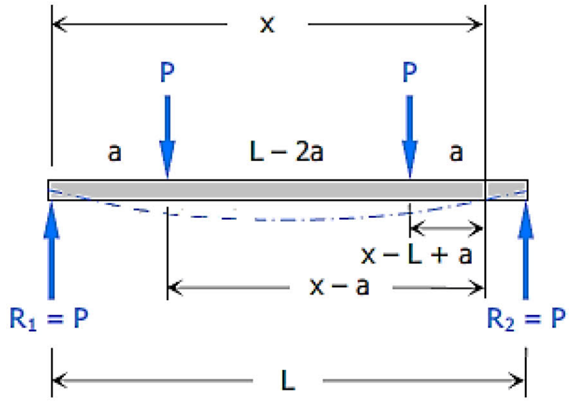
As shown in the figure, a simply supported beam carries two symmetrically placed concentrated loads. Compute the maximum deflection ***v***.



Solution

By symmetry

$R_1 = R_2 = P$



$$EIv'' = Px - P\langle x-a\rangle - P\langle x-L+a\rangle$$

$$EIv' = \frac{1}{2}Px^2 - \frac{1}{2}P\langle x-a\rangle^2 - \frac{1}{2}P\langle x-L+a\rangle^2 + C_1$$

$$EIv = \frac{1}{6}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-L+a\rangle^3 + C_1x + C_2$$

$At\ x = 0,\ v = 0$, therefore $C_2 = 0$

$At\ x = L,\ v = 0$

$$0 = \frac{1}{6}PL^3 - \frac{1}{6}P\langle x-a\rangle^3 + C_1L$$

$$0 = PL^3 - P\left(L^3 - 3L^2a + 3La^2 - a^3\right) - Pa^3 + 6C_1L$$

$$0 = PL^3 - PL^3 + 3PL^2a - 3PLa^2 + Pa^3 - Pa^3 + 6C_1L$$

$$0 = 3PL^2a - 3PLa^2 + 6C_1L \quad \Rightarrow 0 = 3PLa(L-a) + 6C_1L \quad \Rightarrow C_1 = -\frac{1}{2}Pa(L-a)$$

Therefore,

$$EIv = \frac{1}{6}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-L+a\rangle^3 - \frac{1}{2}Pa(L-a)x$$

Maximum deflection will occur at ***x* = ½ *L*** (midspan)

$$EIv_{max} = \frac{1}{6}P\left(\frac{1}{2}L\right)^3 - \frac{1}{6}P\left(\frac{1}{2}L-a\right)^3 - \frac{1}{2}Pa(L-a)\left(\frac{1}{2}L\right)$$

$$EIv_{max} = \frac{1}{48}PL^3 - \frac{1}{6}P\left[\frac{1}{2}(L-2a)\right]^3 - \frac{1}{4}PL^2a + \frac{1}{4}PLa^2$$

$$EIv_{max} = \frac{1}{48}PL^3 - \frac{1}{48}P\left[L^3 - 3L^2(2a) + 3L(2a)^2 - (2a)^3\right] - \frac{1}{4}PL^2a + \frac{1}{4}PLa^2$$

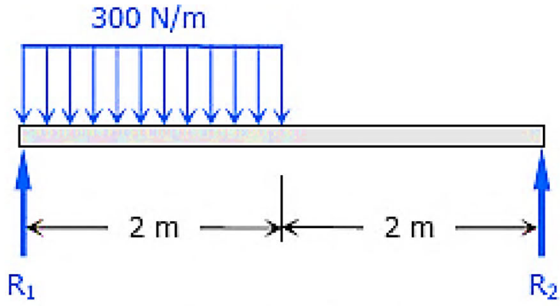
$$EIv_{max} = \frac{1}{48}PL^3 - \frac{1}{48}PL^3 + \frac{1}{8}PL^2a - \frac{1}{4}PLa^2 + \frac{1}{6}Pa^3 - \frac{1}{4}PL^2a + \frac{1}{4}PLa^2$$

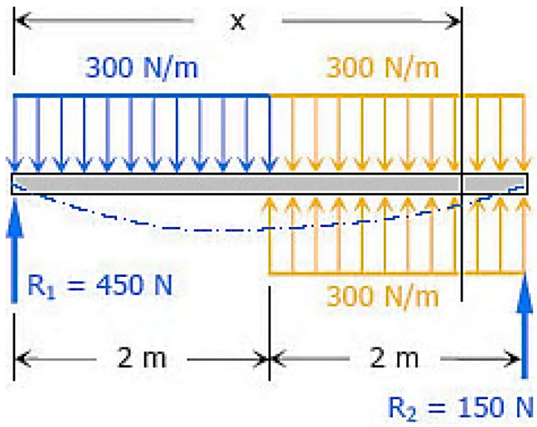
$$EIv_{max} = -\frac{1}{8}PL^2a + \frac{1}{6}Pa^3 \quad \Rightarrow EIv_{max} = -\frac{1}{24}Pa\left(3L^2 - 4a^2\right)$$

$$v_{max} = -\frac{Pa}{24EI}\left(3L^2 - 4a^2\right) \qquad Ans.$$

EXAMPLE 5.3.9

Compute the value of ***EI v*** at midspan for the beam loaded as shown in the figure.





$$\Sigma M_{R2} = 0$$
$$4R_1 = 300(2)(3)$$
$$R1 = 450\text{N}$$
$$\Sigma M_{R1} = 0$$
$$4R_2 = 300(2)(1)$$
$$R_2 = 150\text{N}$$
$$EIv'' = 450x - \frac{1}{2}(300)x^2 + \frac{1}{2}(300)\langle x-2\rangle^2$$
$$EIv'' = 450x - 150x^2 + 150\langle x-2\rangle^2$$
$$EIv' = 225x^2 - 50x^3 + 50\langle x-2\rangle^3 + C_1$$
$$EIv = 75x^3 - 12.5x^4 + 12.5\langle x-2\rangle + C_1x + C_2$$
$$At\ x = 0, \quad v = 0, \text{ therefore } C_2 = 0$$
$$At\ x = 4\,m, \quad v = 0$$
$$0 = 75(4^3) - 12.5(4^4) + 12.5(4-2)^4 + 4C_1 \Rightarrow C_1 = -450 \text{ N.m}^2$$

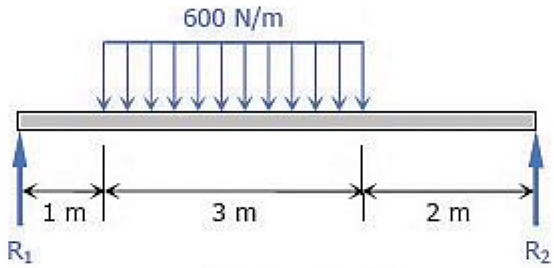
Therefore,

$$EIy = 75x^3 - 12.5x^4 + 12.5\langle x-2\rangle^4 - 450x$$

$$At\ x = 2\,m\ (midspan) \quad \Rightarrow EIy_{midspan} = 75(2^3) - 12.5(2^4) + 12.5(2-2)^4 - 450(2)$$
$$\Rightarrow EIv_{midspan} = -500 \text{ N.m}^3$$

EXAMPLE 5.3.10

Compute the midspan value of **EI δ** for the beam loaded as shown in the figure.



Solution

$\Sigma M_{R2} = 0$

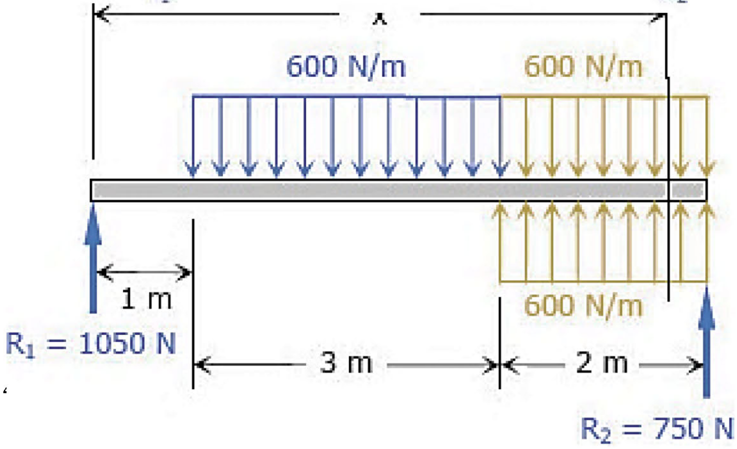
$6R_1 - 600(3)(3.5) = 0$

$R_1 = 1050$ N

$\Sigma M_{R1} = 0$

$-6R_2 + 600(3)(2.5) = 0$

$R_2 = 750$N



$EIv'' = 1050x - \frac{1}{2}(600)\langle x-1\rangle^2 + \frac{1}{2}(600)\langle x -$

$EIv'' = 1050x - 300\langle x-1\rangle^2 + 300\langle x-4\rangle^2$

$EIv' = 525x^2 - 100\langle x-1\rangle^3 + 100\langle x-4\rangle^3 + C_1$

$EIv = 175x^3 - 25\langle x-1\rangle^4 + 25\langle x-4\rangle^4 + C_1x + C_2$

At $x = 0$, $v = 0$, therefore $C_2 = 0$

At $x = 6\ m$, $v = 0$

$0 = 175(6^3) - 25(6-1)^4 + 25(6-4)^4 + 6C_1 \quad \Rightarrow C1 = -3762.5\ \text{N.m}^2$

Therefore,

$EIv = 175x^3 - 25(x-1)^4 + 25(x-4)^4 - 3762.5x$

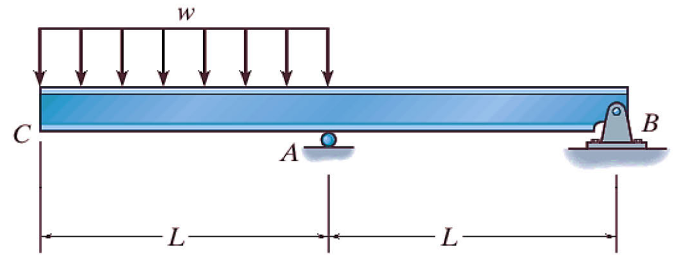
At midspan, $x = 3$ m

$EIv_{midspan} = 175(3^3) - 25(3-1)^4 - 3762.5(3)$

$EIv_{midspan} = -6962.5\ \text{N.m}^3$

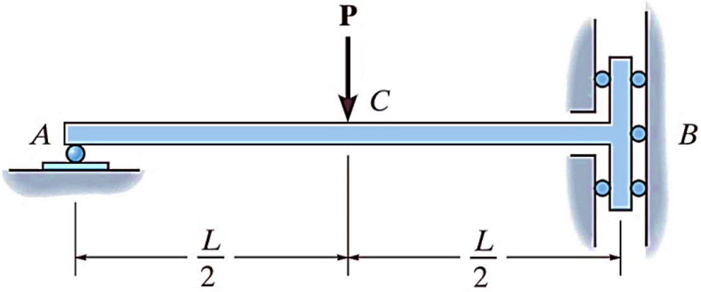
Hw.14

Determine the maximum deflection between the supports ***A*** and ***B. EI*** is constant. Use the method of integration.



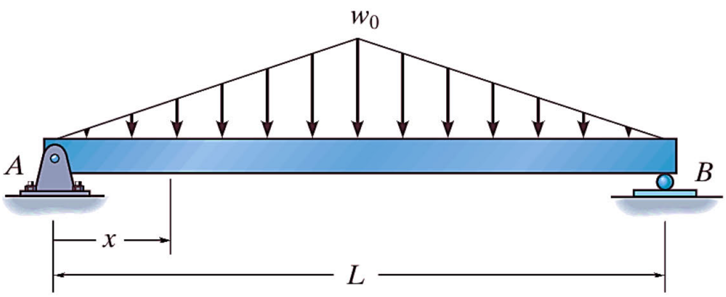
Hw.15

The bar is supported by a roller constraint at ***B***, which allows vertical displacement but resists axial load and moment. If the bar is subjected to the loading shown, determine the slope at ***A*** and the deflection at ***C. EI*** is constant.



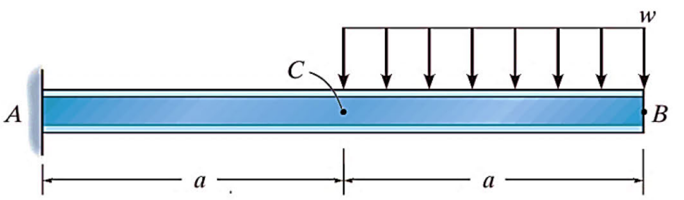
Hw.16

Determine the elastic curve for the simply supported beam using the ***x*** coordinate $\mathbf{0 \le x \le L/2}$ Also, determine the slope at ***A*** and the maximum deflection of the beam. ***EI*** is constant.



Hw.17

Determine the equations of the elastic curve and specify the slope and deflection at ***B*** and ***C. EI*** is constant.



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Hw.18

Determine the equations of the elastic curve, and specify the slope and deflection at point ***B*** and ***C. EI*** is constant.